Minimum Control Power for VTOL Aircraft Stability Augmentation

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Practical design of VTOL aircraft stability augmentation systems based on the state variable methods of modern control theory is illustrated. The approach allows exact realization of desired closed-loop handling qualities in terms of a specified closed-loop characteristic equation. This cannot be done using conventional servoanalysis methods of control system design. The resulting system requires minimum control power levels for aircraft stabilization. The actual amounts of control power are dependent on the atmospheric turbulence environment. A power spectral density method is illustrated which accounts for homogeneous turbulence effects and yields the minimum figure for stabilization control power as a function of gust velocity rms value.

Nomenclature

$CP(t)$ = instantaneous control power
I_x,I_y,I_z = mass moments of inertia about the respective x,y,z axes
j = $(-1)^{\frac{1}{2}}$
L = integral scale of turbulence
L,M,N = roll, pitch, and yaw moments, respectively
s = Laplace complex variable
U_0 = mean wind speed
v,φ,ψ = sideslip velocity, roll angle, and yaw angle, respectively
v_g = lateral component of gust velocity
x_1,x_2,x_3,x_4 = state variables
$\delta_a,\delta_e,\delta_r$ = equivalent aileron, elevator, and rudder deflection angles, respectively
ζ_d,ω_d = "dutch-roll" damping ratio and undamped natural frequency, respectively
φ_c, ψ_c = commanded roll and yaw angles, respectively
$\Phi_{v_g}(s)$ = power spectral density of v_g
σ_{v_g} = root-mean-square value of v_g

Introduction

THE growth of the sprawling megalopolitan areas and the choking ground congestion around present airports make it mandatory for the airline industry to find a far better method of transporting short-haul (less than 500 miles) traffic. As a solution to this problem, many air transportation authorities in industry and government agree that the vertical take off and landing (VTOL) aircraft will be an important mode of transportation by 1980. The Federal Aviation Administration (FAA) is preparing tentative VTOL certification standards. However, much research still needs to be done before quantitative design standards can be written.

The flight control system is a critical part of a VTOL aircraft, particularly in the hover and transition flight modes, where a failure is frequently catastrophic. A number of VTOL crashes have been attributed to control system failures or to a lack of sufficient control power to stabilize the aircraft. Control power is most often defined as the angular acceleration produced by a control input. Instantaneous control power about the pitch, yaw, and roll axes, respectively, is given by

$$CP_y(t) = (1/I_y)(\partial M/\partial \delta_e)\delta_e(t) = M_{\delta_e}\delta_e(t) \quad (1)$$

$$CP_z(t) = (1/I_z)(\partial N/\partial \delta_r)\delta_r(t) = N_{\delta_r}\delta_r(t) \quad (2)$$

$$CP_x(t) = (1/I_x)(\partial L/\partial \delta_a)\delta_a(t) = L_{\delta_a}\delta_a(t) \quad (3)$$

where the control deflections δ_e, δ_r, and δ_a are measured from the trim position for a given flight condition. These equations apply to the stabilization and maneuvering components of control power.

There is a critical need for better methods of determining the minimum levels of control power necessary to provide adequate attitude stabilization and maneuverability in hover and transition. An insufficient amount is unsafe and an excess reduces the available lift engine thrust, as control power is obtained by bleeding air or modulating thrust from the propulsion system. The amount needed for maneuvering is generally independent of aircraft size and dynamic characteristics. However, that needed for stabilization is strongly dependent on the open-loop dynamics, the type and amount of stability augmentation provided, and the turbulence environment, as well as aircraft size. An analytical approach and design methodology is needed which structures the stability augmentation system required for satisfactory aircraft handling qualities, while simultaneously yielding the minimum required values of stabilization control power. This paper describes such an approach.

Some early work on establishing VTOL handling qualities criteria and control power requirements is documented in Refs. 1–5. Representative results of more recent studies are contained in 6–12. However, the research literature and the VTOL aircraft built to date have not recognized the importance of the type of feedback control system used on the resulting control power requirements. Most three-axes stability augmentation systems have employed conventional attitude and rate feedback loops with no regard for what this control law structure means in terms of stabilization control power levels. For example, most vehicles in a hovering mode have nonminimum phase (right half-plane poles and zeros) transfer functions and require unnecessarily high control power levels when stabilized by conventional servoanalysis design techniques. It will be shown herein that modern linear control synthesis methods can be used for direct synthesis of stability augmentation systems yielding prescribed handling qualities and minimum stabilization control power.

Stability Augmentation System Synthesis

Application of new analytical methods is best illustrated by considering simple, specific numerical examples. Those interested in the theoretical developments, proofs, and generaliza-

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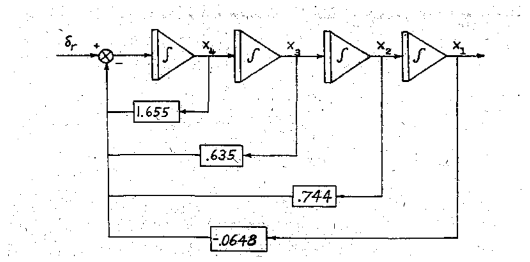

Fig. 1 State variable diagram.

tions of the linear modern control methods can refer to the references cited below.

Consider the task of synthesizing a lateral-directional stability augmentation system for the Doak VZ-4 tilt-duct VTOL aircraft. The flight condition is that of hovering at 100 ft over a fixed ground point in turbulent air with a 35 knot headwind. The Laplace transformed, small-perturbation, lateral-directional equations of motion as determined from Ref. 10 are

$$\begin{matrix}\text{sideslip}\\ \text{yaw}\\ \text{roll}\end{matrix}\begin{bmatrix}(s+0.3) & 0 & \dfrac{-32.2}{s}\\ -0.008 & (s+0.7) & (0.07s-0.06)\\ 0.02 & (0.1s-2) & (s+0.5)\end{bmatrix}\begin{bmatrix}v(s)\\ \dot\psi(s)\\ \dot\varphi(s)\end{bmatrix} =$$

$$\begin{bmatrix}-25 & -2\\ 0.003 & 0.8\\ 0.5 & 0.1\end{bmatrix}\begin{bmatrix}\delta_a(s)\\ \delta_r(s)\end{bmatrix} + \begin{bmatrix}-0.3\\ 0.008\\ -.02\end{bmatrix} v_g(s) \quad (4)$$

The following transfer functions can be computed from Eq. (4):

$$\frac{v(s)}{\delta_r(s)} = \frac{-2s^3 - 2.71s^2 + 0.185s + 54.2}{s^4 + 1.655s^3 + 0.635s^2 + 0.744s - 0.0648} \quad (5)$$

$$\frac{\dot\psi(s)}{\delta_r(s)} = \frac{0.8s^3 + 0.629s^2 + 0.121s + 0.778}{\Delta(s)} \quad (6)$$

$$\frac{\dot\varphi(s)}{\delta_r(s)} = \frac{0.0201s^3 + 1.78s^2 + 5.04s}{\Delta(s)} \quad (7)$$

where $\Delta(s)$ is the same characteristic polynomial as in Eq. (5) and has the factored form in Eq. (8).

$$\Delta(s) = (s - 0.08)(s + 1.59)(s + 0.0725 \pm j0.715) \quad (8)$$

The complex roots yield the "dutch-roll" quadratic with $\zeta_d = 0.101$ and $\omega_d = 0.717$ rad/sec.

The fourth-order system described by the three transfer functions can be reduced to an equivalent set of 4 first-order state variable differential equations by use of the state variable diagram of Fig. 1.[13] It is determined from Fig. 1 that

$$\dot x_1 = x_2, \quad \dot x_2 = x_3, \quad \dot x_3 = x_4,$$
$$\dot x_4 = 0.0648x_1 - 0.744x_2 - 0.635x_3 - 1.655x_4 + \delta_r \quad (9)$$

Note that the coefficients in the last equation are the negative of those in $\Delta(s)$. In matrix form, Eq. (9) becomes

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0.0648 & -0.744 & -0.635 & -1.655\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \delta_r\end{bmatrix} \quad (10)$$

The actual output variables are related to the state variables by Eq. (11), where the coefficient 3 × 4 matrix is composed of the numerator coefficients in Eqs. (5–7).

$$\begin{bmatrix}v\\ \dot\psi\\ \dot\varphi\end{bmatrix} = \begin{bmatrix}54.2 & 0.185 & -2.71 & -2\\ 0.778 & 0.121 & 0.629 & 0.8\\ 0 & 5.04 & 1.78 & 0.0201\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (11)$$

It is well known in modern linear control theory that a single control input (δ_r in this case) can achieve any desired set of closed-loop poles if all state variables are sensed and fed back, and the resulting closed-loop characteristic equation will be the same order as that for the open-loop (fourth-order in this case).[14] Furthermore, the control law is of the form in Eq. (12) and is optimal in that the weighted integral of δ_r^2 is a minimum.

$$\delta_r = -k_1x_1 - k_2x_2 - k_3x_3 - k_4x_4 = -[k_1, k_2, k_3, k_4]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (12)$$

where the k's are numerical gains. Substituting Eq. (12) into Eq. (10) gives

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ (0.0648 - k_1) & (-0.744 - k_2) & (-0.635 - k_3) & (-1.655 - k_4)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (13)$$

The last row elements in Eq. (13) are now the coefficients of the closed-loop characteristic polynomial.

To further the illustration, suppose that desirable handling qualities in this flight condition required the following closed-loop characteristic polynomial:

$$(s + 0.3)(s + 1)(s^2 + 3s + 9) = s^4 + 4.3s^3 + 13.2s^2 + 12.6s + 2.7 \quad (14)$$

The quadratic gives a closed-loop "dutch-roll" damping ratio of 0.5 and natural frequency of 3 rad/sec. Therefore, from Eq. (13) it follows that

$$0.0648 - k_1 = -2.7, \quad -0.744 - k_2 = -12.6,$$
$$-0.635 - k_3 = -13.2, \quad -1.665 - k_4 = -4.3 \quad (15)$$

Thus, solving Eq. (15) for the k's, Eq. (12) becomes

$$\delta_r = -[2.765, 11.856, 12.565, 2.645]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (16)$$

Since sensors can, in general, measure only output variables and not individual state variables, it is necessary to express Eq.

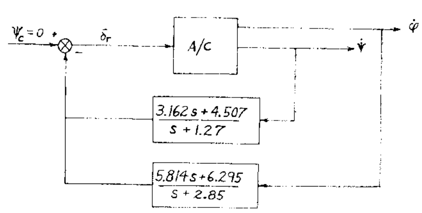

Fig. 2 Feedback system.

(16) in terms of the actual outputs. Equation (11) expresses the outputs in terms of the states, but the coefficient matrix is not square and has no inverse. To solve for the states, a fourth output variable in terms of the states is needed. Various possibilities exist; one could use v, $\dot{\psi}$, or $\dot{\varphi}$. However, this would require sensing and feeding back 4 outputs. A procedure exists whereby fewer than 4 outputs can be sensed if use is made of left half-plane zeros of Eqs. (6) and (7) to construct physically realizable, passive filters in the feedback paths.[15]

The numerators of Eqs. (6) and (7) can be factored as follows:

$$\frac{\dot{\psi}(s)}{\delta_r(s)} = \frac{(s + 1.27)(0.8s^2 - 0.386s + 0.612)}{\Delta(s)} \tag{17}$$

$$\frac{\dot{\varphi}(s)}{\delta_r(s)} = \frac{(s + 2.85)(0.02s^2 + 1.72s)}{\Delta(s)} \tag{18}$$

$$\frac{\dot{\psi}(s)}{\delta_r(s)} \cdot \frac{1}{s + 1.27} = \frac{0.8s^2 - 0.386s + 0.612}{\Delta(s)} \tag{19}$$

$$\frac{\dot{\varphi}(s)}{\delta_r(s)} \cdot \frac{1}{s + 2.85} = \frac{0.02s^2 + 1.72s}{\Delta(s)} \tag{20}$$

In a manner analogous to the way the coefficients were obtained in Eq. (11), we can write

$$\begin{bmatrix} \dot{\psi}(s) \\ \dfrac{\dot{\psi}(s)}{s + 1.27} \\ \dot{\varphi}(s) \\ \dfrac{\dot{\varphi}(s)}{s + 2.85} \end{bmatrix} = \begin{bmatrix} 0.778 & 0.121 & 0.629 & 0.8 \\ 0.612 & -0.386 & 0.8 & 0 \\ 0 & 5.04 & 1.78 & 0.02 \\ 0 & 1.72 & 0.02 & 0 \end{bmatrix} \begin{bmatrix} x_1(s) \\ x_2(s) \\ x_3(s) \\ x_4(s) \end{bmatrix} \tag{21}$$

Solving Eq. (21) for the state vector gives

$$\begin{bmatrix} x_1(s) \\ x_2(s) \\ x_3(s) \\ x_4(s) \end{bmatrix} = \begin{bmatrix} 0.023 & 1.606 & -0.758 & 2.579 \\ 0 & 0 & -0.008 & 0.602 \\ -0.0145 & 0.0185 & 0.577 & -1.685 \\ 1.24 & -1.58 & 0.285 & -1.277 \end{bmatrix} \begin{bmatrix} \dot{\psi}(s) \\ \dfrac{\dot{\psi}(s)}{s + 1.27} \\ \dot{\varphi}(s) \\ \dfrac{\dot{\varphi}(s)}{s + 2.85} \end{bmatrix} \tag{22}$$

Combining Eqs. (16) and (22),

$$\delta_r(s) = -[3.162, 0.492, 5.814, -10.255] \begin{bmatrix} \dot{\psi}(s) \\ \dfrac{\dot{\psi}(s)}{s + 1.27} \\ \dot{\varphi}(s) \\ \dfrac{\dot{\varphi}(s)}{s + 2.85} \end{bmatrix} \tag{23}$$

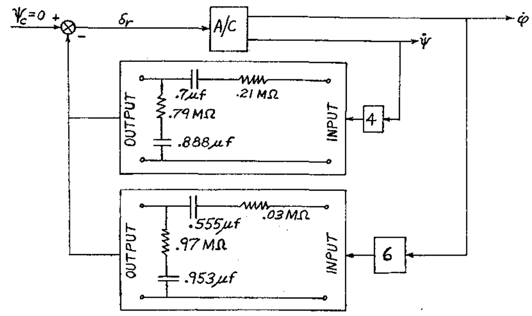

Fig. 3 A passive filter realization with δ_r control.

or

$$\delta_r(s) = -\left[\frac{3.162s + 4.507}{s + 1.27}\right]\dot{\psi}(s) - \left[\frac{5.814s + 6.295}{s + 2.85}\right]\dot{\varphi}(s) \tag{24}$$

Equation (24) is the control law which yields the desired closed-loop dynamics of Eq. (14) and requires only yaw rate and roll rate sensing. Figure 2 shows the system schematic.

The feedback transfer functions can be realized as first-order passive filters utilizing resistors and capacitors along with gain amplifiers. Such realizations are not unique in terms of values on the electronic elements. One practical realization, shown in Fig. 3, can be obtained by writing Eq. (24) in the following form:

$$\delta_r(s) = -4\left[\frac{0.79 \times 10^6 + 1/0.888 \times 10^{-6}\, s}{10^6 + 1/0.788 \times 10^{-6}\, s}\right]\dot{\psi}(s) - 6\left[\frac{0.97 \times 10^6 + 1/0.953 \times 10^{-6}\, s}{10^6 + 1/0.351 \times 10^{-6}\, s}\right]\dot{\varphi}(s) \tag{25}$$

Reference 16 can be consulted for details to go from Eq. (25) to the element values in Fig. 3. The input $\dot{\psi}_c = 0$ indicates a regulator type of augmentation system, where it is desired to maintain zero perturbations from the trim conditions represented by the "frozen point" equations of motion.

Control Power Requirements

In order to determine minimum required values of stabilization control power, the effects of atmospheric homogeneous and heterogeneous turbulence must be considered. Homogeneous turbulence refers to that which can be described in a statistical sense through use of power spectral density (PSD) techniques and heterogeneous or discrete turbulence refers to vortex patterns generated by obstacles such as trees, hills, buildings, etc. The levels of control power needed to counteract disturbances to a VTOL aircraft flying on the lee side of obstacles, where the predominant effect is discrete turbulence, can be analyzed using momentum transfer concepts.[17] Effects of homogeneous turbulence will be considered here.

By disregarding gust velocity spatial distribution effects, which are important when making an accurate analysis,[18] only the lateral component of gust velocity v_g excites lateral-directional responses. Thus, through use of Eq. (23), the system diagram can be drawn as in Fig. 4. Using Mason's method[19] for closed-loop transfer function determination, it is determined from Fig. 4 that

$$\delta_r(s) = \frac{\left[-3.162\,\dfrac{\dot{\psi}}{v_g} - 5.814\,\dfrac{\dot{\varphi}}{v_g}\right] v_g(s)}{1 + 3.162\,\dfrac{\dot{\psi}}{\delta_r} + 0.492\,\dfrac{\dot{\psi}}{\delta_r(s + 1.27)} + 5.814\,\dfrac{\dot{\varphi}}{\delta_r} - 10.255\,\dfrac{\dot{\varphi}}{\delta_r(s + 2.85)}} \tag{26}$$

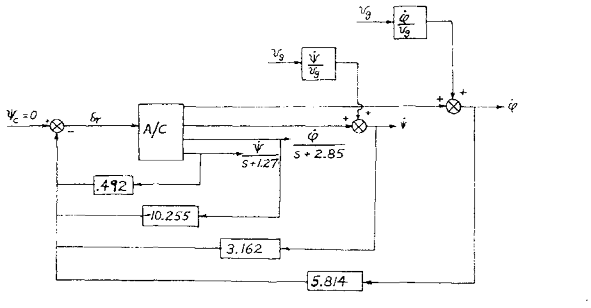

Fig. 4 Feedback system with lateral turbulence.

The denominator of Eq. (26) must be of fourth-order, and substitution of Eqs. (6, 7, 19, and 20) into (26) will yield a denominator given by Eq. (14)—the closed-loop characteristic polynomial. The numerator transfer functions as obtained from Eq. (4) are

$$\frac{\dot\psi(s)}{v_g(s)} = \frac{0.009s^3 + 0.003s^2}{\Delta(s)} \tag{27}$$

$$\frac{\dot\varphi(s)}{v_g(s)} = \frac{-0.021s^3 - 0.054s^2 + 0.0054s}{\Delta(s)} \tag{28}$$

Therefore, Eq. (26) becomes

$$\delta_r(s) = \frac{0.093s^3 + 0.307s^2 - 0.032s}{s^4 + 4.3s^3 + 13.2s^2 + 12.6s + 2.7} v_g(s) \tag{29}$$

It is well known from stochastic analysis that the power spectral density of δ_r is

$$\Phi_{\delta_r}(s) = \left|\frac{0.093s^3 + 0.307s^2 - 0.032s}{s^4 + 4.3s^3 + 13.2s^2 + 12.6s + 2.7}\right|^2 \Phi_{v_g}(s) \tag{30}$$

A mathematical model for v_g sufficiently accurate for our purpose is[20]

$$\Phi_{v_g}(s) = 2\sigma^2_{v_g} \frac{L}{U_0} \frac{1}{[1 - (Ls/U_0)^2]} \tag{31}$$

where

$$\sigma^2_{v_g} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \Phi_{v_g}(s)\, ds \tag{32}$$

Equation (31) is based on the assumptions of stationary, homogeneous, isotropic turbulence with a Gaussian amplitude probability density. The common value for the integral scale L for low altitude turbulence is $L = 0.9$ (altitude). Therefore, $L = 90$ ft, $U_0 = 35$ knots $= 59$ fps, and

$$\Phi_{v_g}(s) = \sigma^2_{v_g} \frac{1.314}{(s + 0.657)(-s + 0.657)} \tag{33}$$

The mean square value of δ_r is given by

$$\sigma^2_{\delta_r} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \Phi_{\delta_r}(s) ds \tag{34}$$

Substituting Eqs. (30) and (33) into Eq. (34) and integrating with the aid of integral tables[21] yields

$$\sigma_{\delta_r}{}^2 = 0.0012\sigma_{v_g}{}^2 \tag{35}$$

From Eq. (2), the value of root-mean-square (rms) control power is

$$CP_{rms} = N_{\delta_r}\sigma_{\delta_r} = (0.8)(0.0347\sigma_{v_g}) = 0.0278\sigma_{v_g} \tag{36}$$

If one were to specify that the available control power be the "three-sigma" value given by Eq. (37), this would mean that the probability of the instantaneous required control power exceeding that available is 0.0027. In other words, 99.73% of the time the amount given by Eq. (37) would be sufficient for stabilization purposes.

$$CP_{\text{available}} = 3N_{\delta_r}\sigma_{\delta_r} = 0.0834\sigma_{v_g} \tag{37}$$

In a mean wind of 59 fps, a reasonable upper limit on severe turbulence intensity (rms value σ_{v_g}) would be about 14 fps. Thus, in this example available yaw control power of 1.168 rad/sec^2 would be a safe, but not overly conservative, value. This minimum safe value has been obtained by taking into account the aircraft open-loop dynamics, the stability augmentation which yields desired handling qualities, and the homogeneous turbulence.

Aileron Control

The above stabilization system used an equivalent rudder input δ_r as the control input. The lateral-directional stability augmentation can also be accomplished by use of equivalent aileron control. The synthesis procedure is exactly the same. Equations (13) and (15) still apply and Eq. (16) becomes

$$\delta_a = -[2.765, 11.856, 12.565, 2.645] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{38}$$

Employing the $\dot\psi/\delta_a$ and $\dot\varphi/\delta_a$ transfer functions, the equation analogous to Eq. (21) is

$$\begin{bmatrix} \dot\psi(s) \\ \dfrac{\dot\psi(s)}{s + 2.06} \\ \dot\varphi(s) \\ \dfrac{\dot\varphi(s)}{s + 0.48} \end{bmatrix} = \begin{bmatrix} 0.1315 & -0.0568 & -0.214 & -0.0322 \\ 0.1287 & -0.181 & -0.0322 & 0 \\ 0 & 1.025 & 2.39 & 0.503 \\ 0 & 2.15 & 0.503 & 0 \end{bmatrix} \begin{bmatrix} x_1(s) \\ x_2(s) \\ x_3(s) \\ x_4(s) \end{bmatrix} \tag{39}$$

The equations analogous to Eqs. (24) and (25) are Eqs. (40) and (41), respectively,

$$\delta_a(s) = -\left[\frac{24.73s + 49.59}{s + 2.06}\right] \dot\psi(s) - \left[\frac{6.62s + 5.99}{s + 0.48}\right] \dot\varphi(s) \tag{40}$$

$$\delta_a(s) = -50 \left[\frac{0.495 \times 10^6 + 1/1.007 \times 10^{-6}s}{10^6 + 1/0.485 \times 10^{-6}s}\right] \dot\psi(s) - 15 \left[\frac{0.441 \times 10^6 + 1/2.5 \times 10^{-6}s}{10^6 + 1/2.085 \times 10^{-6}s}\right] \dot\varphi(s) \tag{41}$$

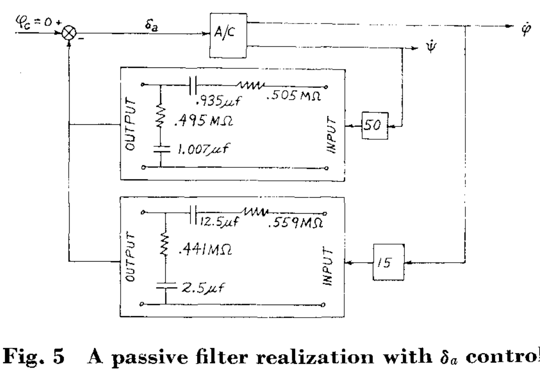

Fig. 5 A passive filter realization with δ_a control.

A physical realization of Eq. (41) is shown in Fig. 5. Aileron control requires considerably higher gains, in this case, than does rudder control (50 and 15 as opposed to 4 and 6). In general, high gains are undesirable; however, aileron control power must be examined before comparing the two systems.

The equation similar to Eq. (30) is

$$\Phi_{\delta_a}(s) = \left|\frac{-0.086s^3 + 0.29s^2 - 0.036s}{s^4 + 4.3s^3 + 13.2s^2 + 12.6s + 2.7}\right|^2 \Phi_{v_g}(s) \quad (42)$$

Substituting Eq. (33) into Eq. (42) and integrating Eq. (43) yields Eq. (44)

$$\sigma_{\delta_a}{}^2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \Phi_{\delta_a}(s)ds \quad (43)$$

$$\sigma_{\delta_a}{}^2 = 0.00096\sigma_{v_g}{}^2 \quad (44)$$

From Eq. (3),

$$CP_{rms} = L_{\delta_a}\sigma_{\delta_a} = (0.5)(0.031\sigma_{v_g}) = 0.0155\sigma_{v_g} \quad (45)$$

The "three-sigma" value is

$$CP_{\text{available}} = 3L_{\delta_a}\sigma_{\delta_a} = 0.0465\sigma_{v_g} \quad (46)$$

For $\sigma_{v_g} = 14$ fps, the available roll control power should be 0.651 rad/sec^2 to accomplish the desired stabilization. This is less than the 1.168 rad/sec^2 needed with rudder control; and if the choice were to be made solely on the basis of control power, one would be tempted to choose aileron control. However, more direct indicators than control power of the amount of engine bleed air needed are σ_{δ_r} and σ_{δ_a}. Control power is expressed as angular acceleration about the yaw or roll axes and is a strong function of the control sensitivity terms, N_{δ_r} and L_{δ_a}. N_{δ_r} and L_{δ_a} are time independent and are dependent on the geometry of the control configuration, i.e., location of the points of application of the modulated gas streams being used for control.

Consider, as an example, reaction jets located at the wing tips for roll control. The control moment of one jet would be the semispan b times the reaction force of the gas stream. Instantaneous control power would be the control moment divided by I_x. The reaction force is

$$\text{force} = \tfrac{1}{2}\rho V^2 A \quad (47)$$

and

$$CP_x(t) = L_{\delta_a}\delta_a(t) = \frac{\frac{1}{2}\rho V^2 Ab}{I_x} \quad (48)$$

where ρ is the gas density, V the gas velocity at the nozzle exit, and A the nozzle area. Therefore, δ_a and thus σ_{δ_a} are proportional to the mass flow rate (ρVA) of engine bleed gas used for reaction control.

In our example, $\sigma_{\delta_r} = 0.0347\sigma_{v_g}$ and $\sigma_{\delta_a} = 0.031\sigma_{v_g}$. The amount of bleed gas used for stabilization is nearly the same for both systems. The choice might well be the δ_r control system, since it requires lower gains than the δ_a system. But on a minimum control power basis, which has been shown to be misleading, δ_a would be the choice.

Conclusions

A practical design procedure, utilizing the state variable formulations of linear modern control theory, for synthesis of VTOL aircraft stability augmentation systems has been illustrated. If the aircraft handling qualities can be specified in terms of a desired closed-loop characteristic equation, this equation can be realized exactly, which cannot be done when using conventional servoanalysis methods of control system synthesis.

Factors which greatly influence minimum levels of stabilization control power are aircraft open-loop dynamics, type of stability augmentation, and the turbulence environment. The design procedure put forth, logically and systematically considers these factors and structures the stability augmentation system which requires minimum engine bleed gas mass flow rates to achieve a safe level of controllability.

The methods illustrated could just as readily be applied to the design of longitudinal stability augmentation systems with equivalent elevator deflection as the control input.

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